

Optimal control and design of flexible multibody systems

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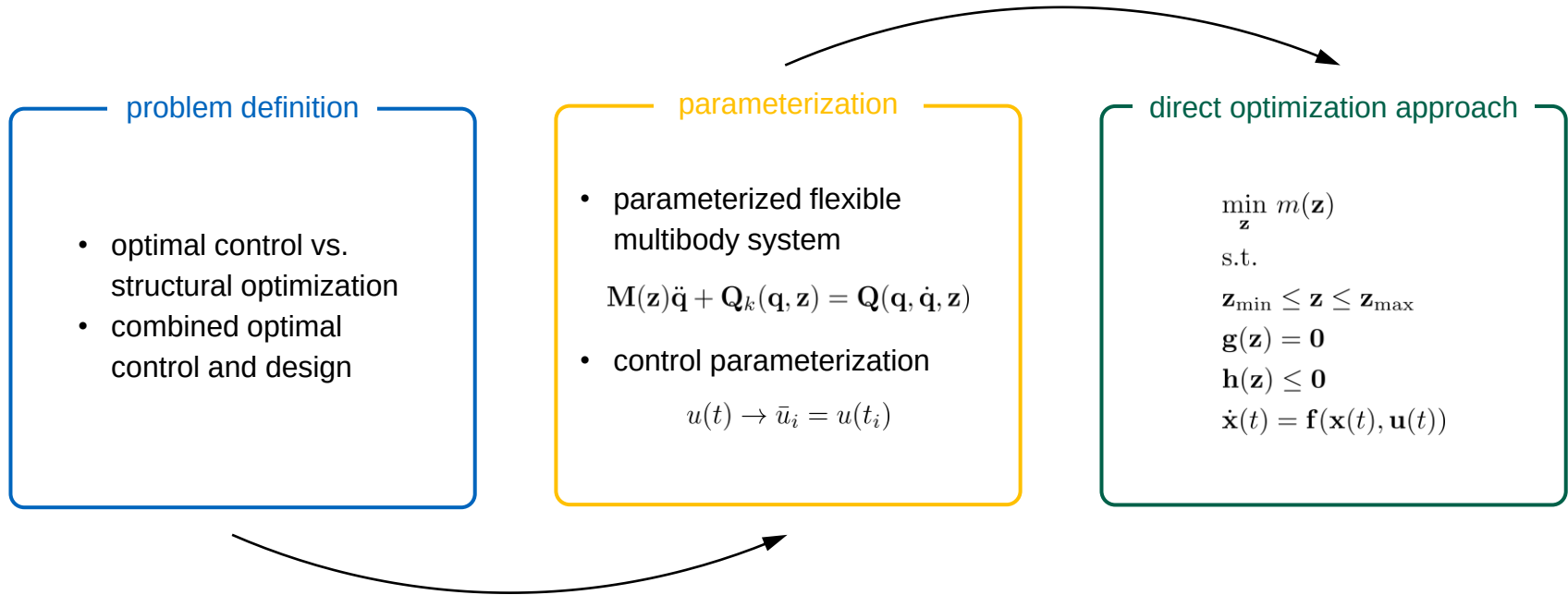
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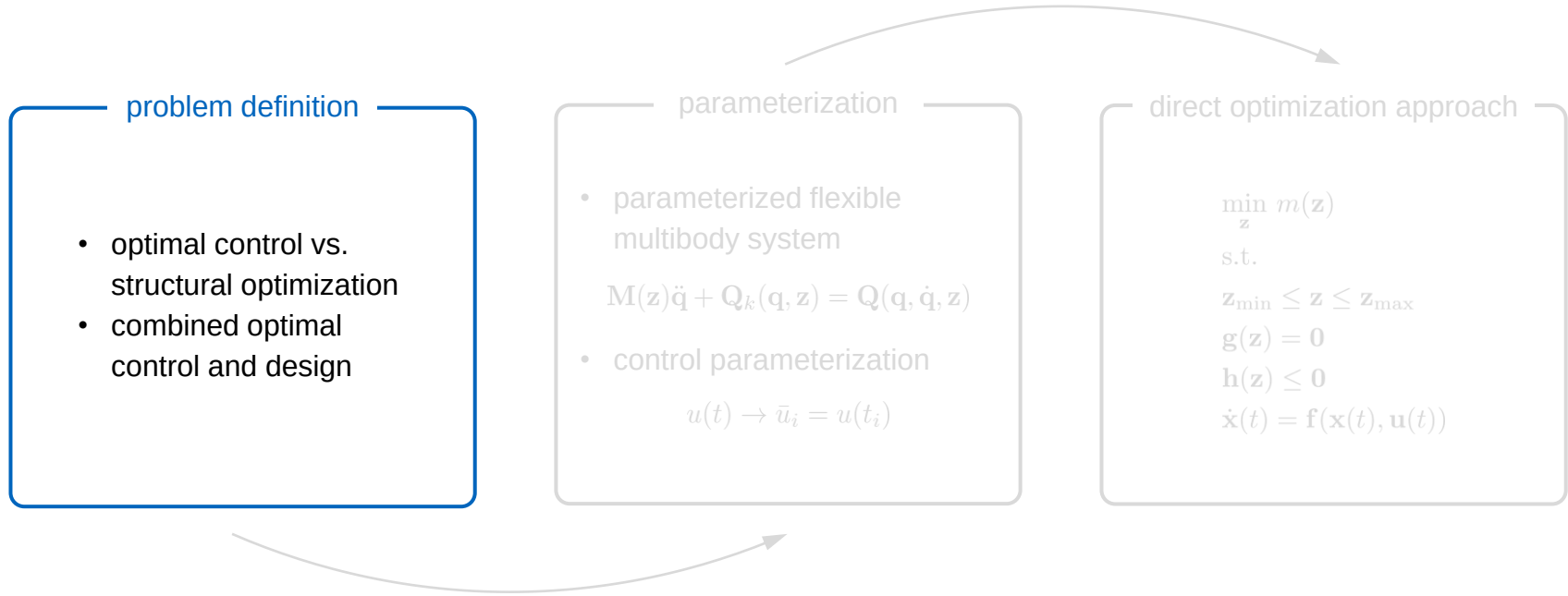
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Outline



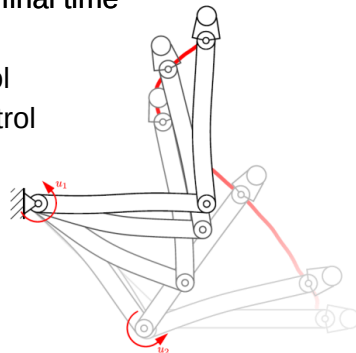
Outline



Optimal Control vs. Structural Optimization

Optimal Control

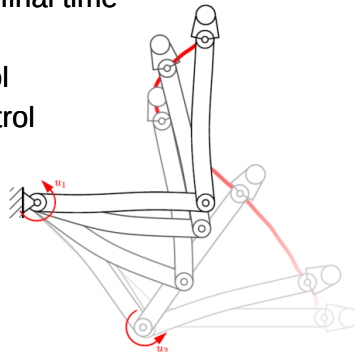
- Given:
 - Geometric parameters of a dynamical system
 - Constraints
- Optimization variables:
 - Control history incl. final time
- Examples:
 - Time-optimal control
 - Energy optimal control



Optimal Control vs. Structural Optimization

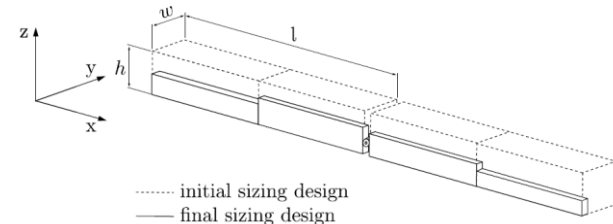
Optimal Control

- Given:
 - Geometric parameters of a dynamical system
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- Examples:
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Structural Optimization

- Given:
 - Control history incl. final time
 - Constraints
- Optimization variables:
 - Geometric parameters of a dynamical system
- Examples:
 - Mass minimization
 - Compliance minimization

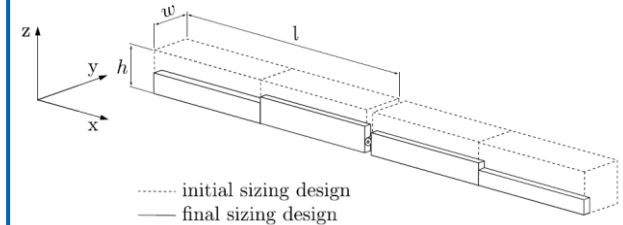
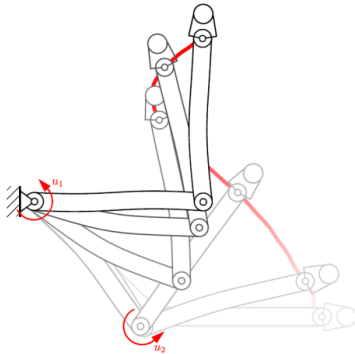


Optimal Control and Design

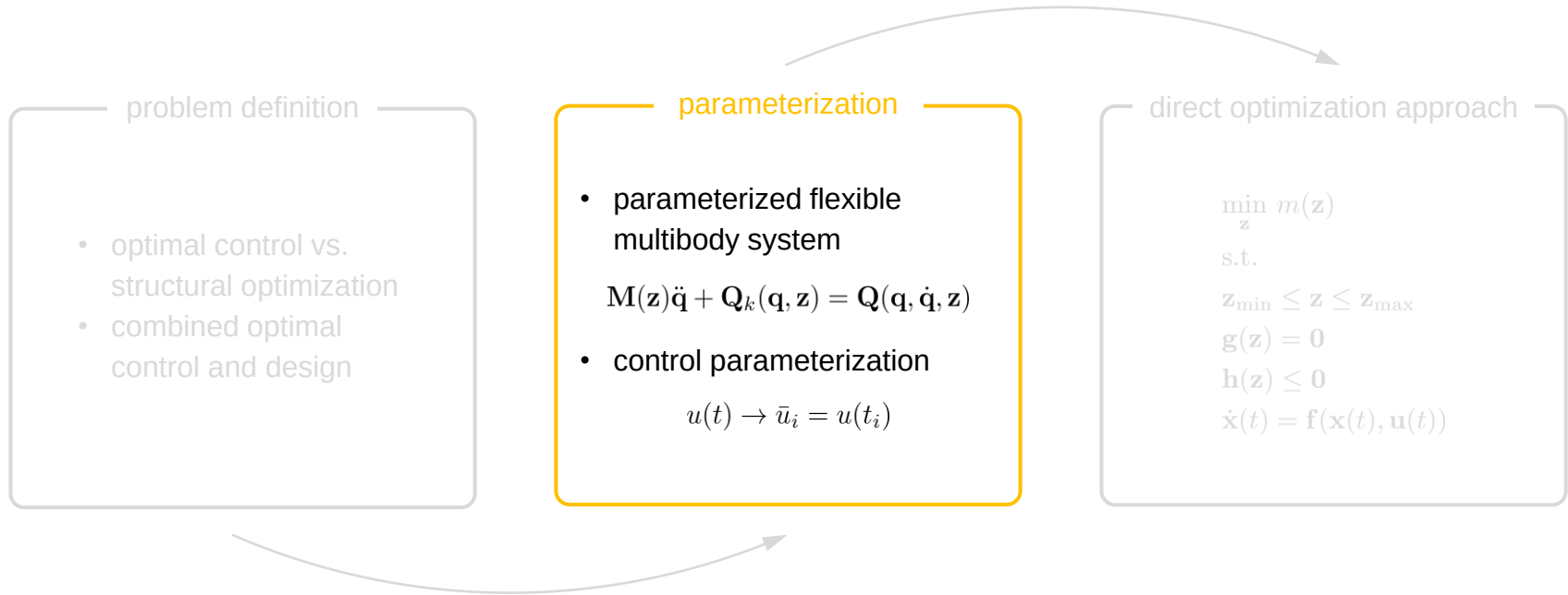
optimal control and design

problem definition

- Given:
 - Constraints
- Optimization variables:
 - Geometric parameters of a dynamical system
 - Control history incl. final time
- Examples:
 - Time-optimal control
 - Energy optimal control
 - Mass minimization
 - Compliance minimization

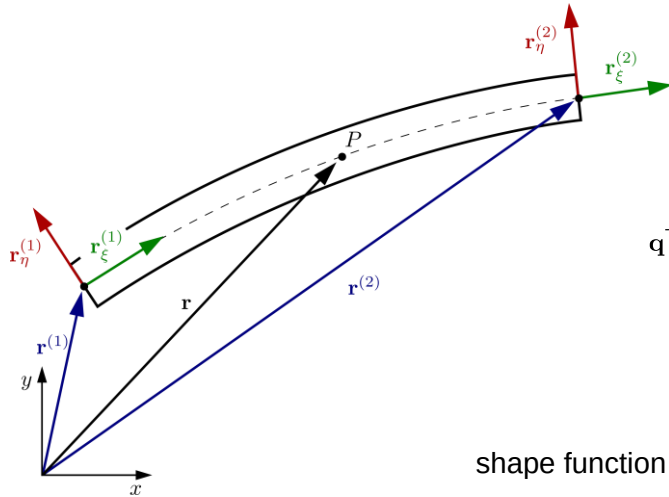


Outline



ANCF Beam Element: Kinematics

Following Omar and Shabana [1]



$$\mathbf{q}^T = \left(\mathbf{r}^{(1)T}, \mathbf{r}_\xi^{(1)T}, \mathbf{r}_\eta^{(1)T}, \mathbf{r}^{(2)T}, \mathbf{r}_\xi^{(2)T}, \mathbf{r}_\eta^{(2)T} \right)$$

$$\mathbf{r} = \mathbf{S}\mathbf{q}$$

shape function matrix $\mathbf{S}$ position vector $\mathbf{r}$ degrees of freedom $\mathbf{q}$

key facts

- ✱ shear deformable
- ✱ angle free formulation
- ✱ allows large rotations and deformations
- ✱ constant mass matrix
- ✱ complex nonlinear force

[1] Omar, A.M. and Shabana, A.A.: A TWO-DIMENSIONAL SHEAR DEFORMABLE BEAM FOR LARGE ROTATION AND DEFORMATION PROBLEMS. Journal of Sound and Vibration 243(3), 565-576 (2001). DOI 10.1006/jsvi.2000.3416

ANCF Beam Element: Generalized Forces

Strain energy following Gerstmayr et al. [2]:

$$U = \frac{1}{2} w \int_{-1/2}^{1/2} \int_0^1 \underbrace{\boldsymbol{\varepsilon}^T \mathbf{D}^0 \boldsymbol{\varepsilon}}_{\text{matrix of elasticities}} J \, d\alpha d\beta + \frac{1}{2} w h \int_0^1 \underbrace{\boldsymbol{\varepsilon}^T \mathbf{D}^\nu \boldsymbol{\varepsilon}}_{\text{engineering strains}} J \, d\alpha$$

$\xrightarrow{\hspace{10em}}$
 $\mathbf{Q}_k = \left(\frac{\partial U}{\partial \mathbf{q}} \right)^T$

$\underbrace{\hspace{10em}}_{\text{determinante of deformation gradient}}$

Virtual work due to a torque M_i :

$$\delta W_i = M_i \delta \theta = M_i \frac{\partial \theta}{\partial \mathbf{q}} \delta \mathbf{q}$$

$\underbrace{\hspace{10em}}_{\text{angle of rotation of the cross-section}}$

$$\mathbf{Q}_u = u \left(\frac{\partial \theta}{\partial \mathbf{q}} \right)^T$$

$$\mathbf{Q}_d = -d\dot{\theta} \left(\frac{\partial \theta}{\partial \mathbf{q}} \right)^T$$

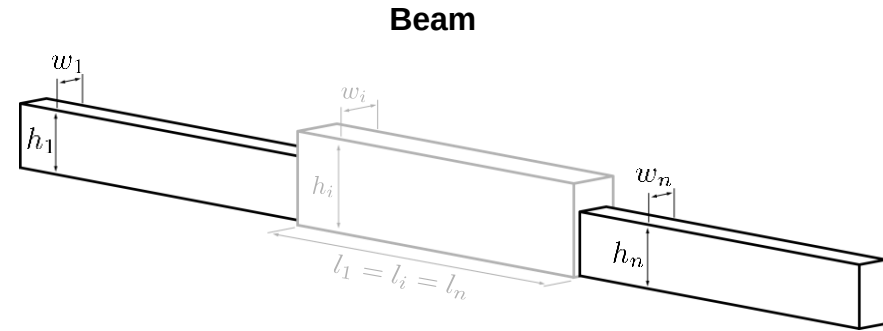
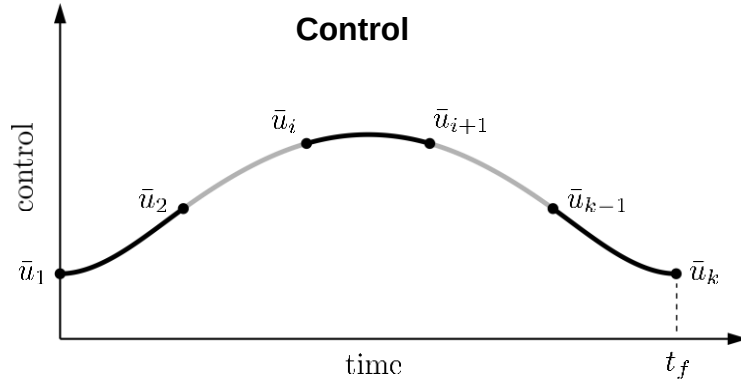
equations of motion

$$\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{v}} \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_u + \mathbf{Q}_d - \mathbf{Q}_k \end{pmatrix}$$

generalized velocities: $\mathbf{v} = \dot{\mathbf{q}}$

[2] Gerstmayr, J., Matikainen, M.K. and Mikkola, A.M.: *A geometrically exact beam element based on the absolute nodal coordinate formulation*. Multibody System Dynamics 20(4), 359–384 (2008). DOI 10.1007/s11044-008-9125-3

Control and Beam Parameterization



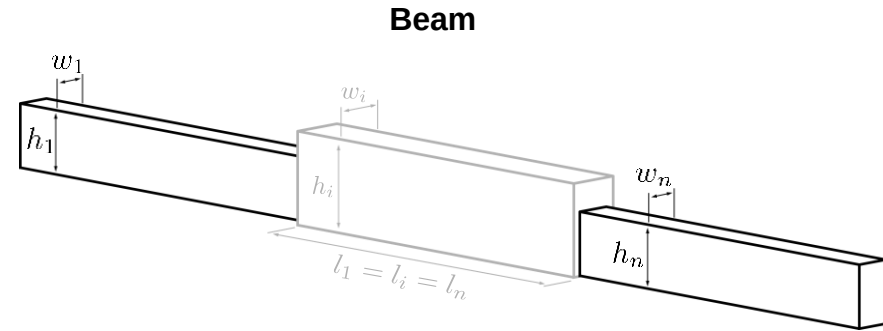
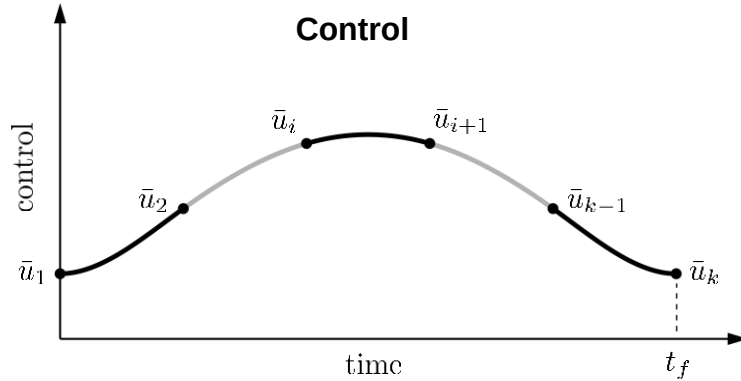
interpolation function $\lceil$

$$u(t) = \mathbf{C}(t) \begin{pmatrix} \bar{u}_1 \\ \vdots \\ \bar{u}_k \end{pmatrix} = \mathbf{C}(t) \bar{\mathbf{u}}$$

$\lfloor$ grid nodes

$$\mathbf{p} = \begin{pmatrix} h_1 \\ \vdots \\ h_n \\ w_1 \\ \vdots \\ w_n \end{pmatrix} \begin{matrix} \text{— height} \\ \\ \text{— width} \end{matrix}$$

Control and Beam Parameterization



interpolation function $\lceil$

$$u(t) = \mathbf{C}(t) \begin{pmatrix} \bar{u}_1 \\ \vdots \\ \bar{u}_k \end{pmatrix} = \mathbf{C}(t) \bar{\mathbf{u}}$$

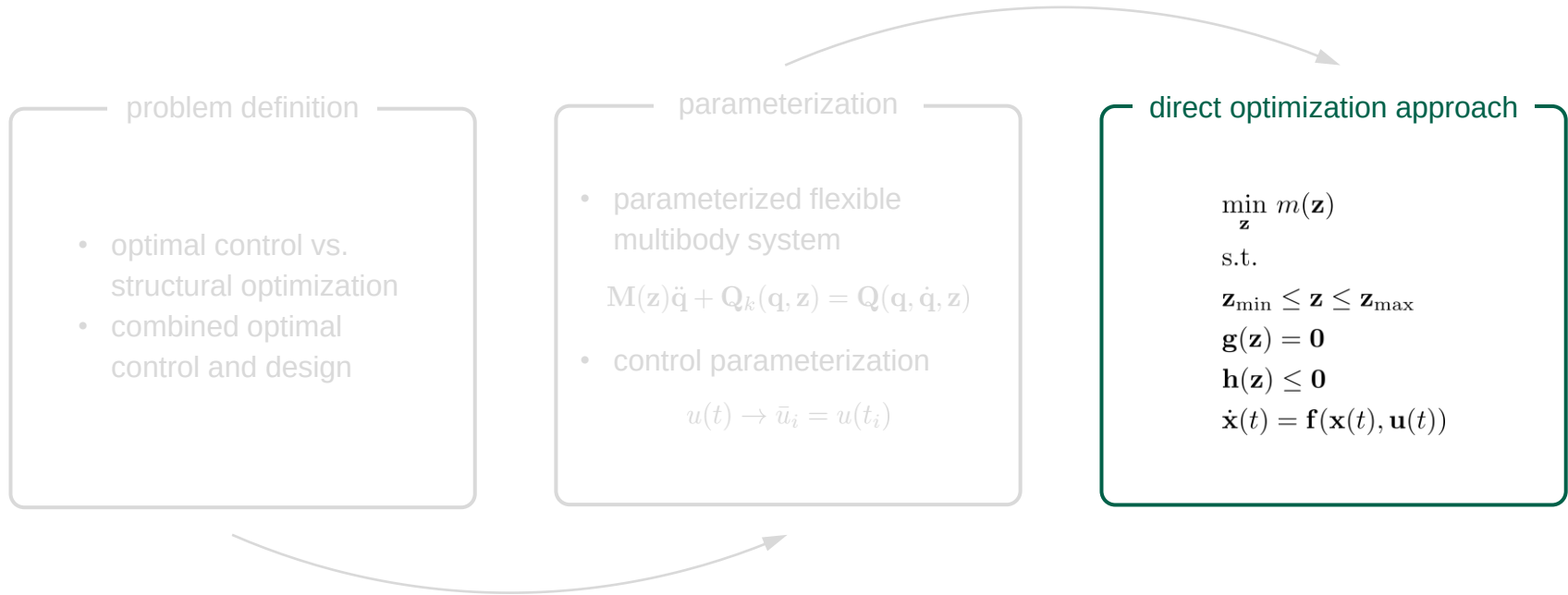
$\lfloor$ grid nodes

optimization variables

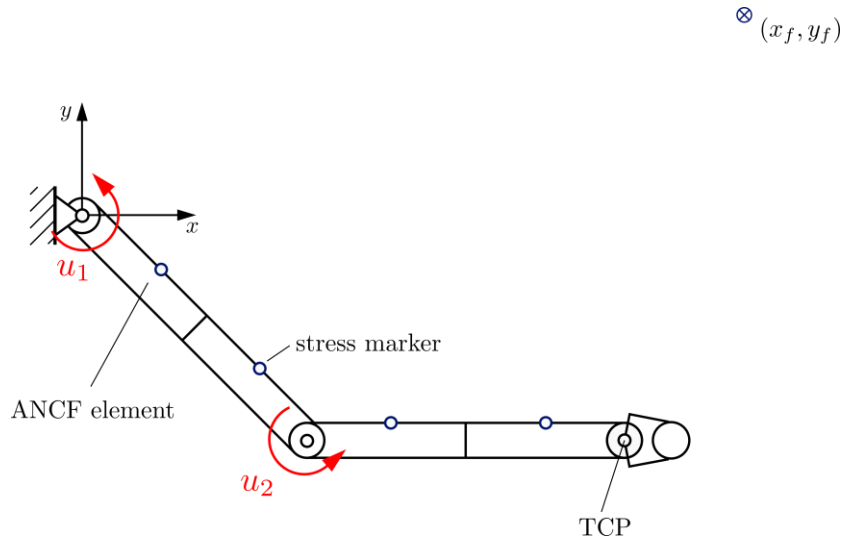
$$\mathbf{z} = \begin{pmatrix} \mathbf{p} \\ \bar{\mathbf{u}} \end{pmatrix}$$

$$\mathbf{p} = \begin{pmatrix} h_1 \\ \vdots \\ h_n \\ w_1 \\ \vdots \\ w_n \end{pmatrix} \begin{matrix} \text{— height} \\ \\ \text{— width} \end{matrix}$$

Outline



Example: Flexible Two Arm Manipulator



optimization problem

$$\min_{\mathbf{z}} m(\mathbf{z})$$

s.t.

$$\mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max}$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

$$\mathbf{x}(t_f) = \mathbf{x}_f$$

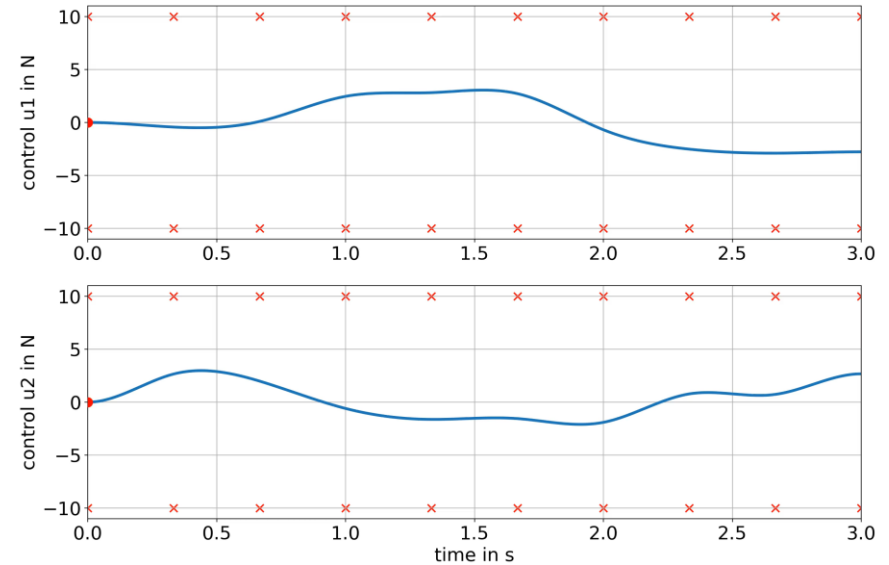
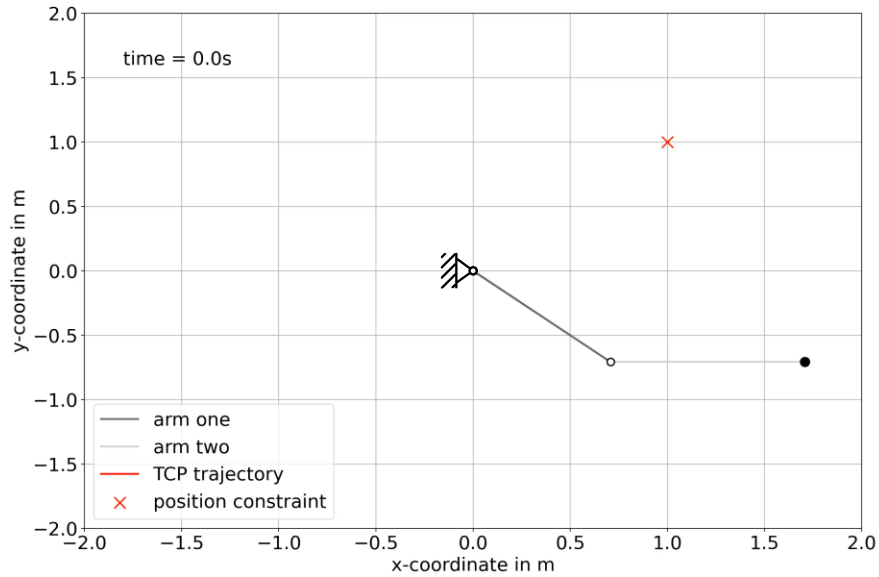
$$\sigma_{V,i}(t)/\sigma_{V,\max} \leq 1 \quad i \in \{1, 2, \dots, N_V\}$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$$

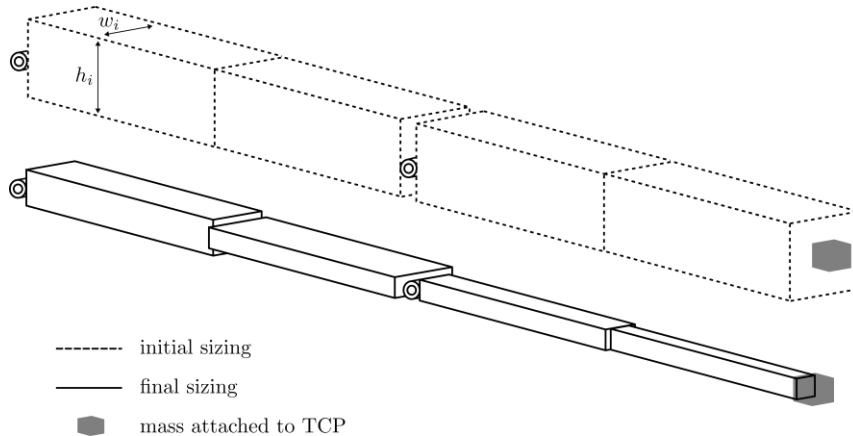
optimization algorithms

- IP-approach (IPOPT)
- using finite-differences

Optimization Results: Control



Optimization Results: Design



final sizing of beam elements

index i	height h_i^* in mm	width w_i^* in mm
1	1.38	2.34
2	1.00	2.72
3	1.00	1.57
4	1.00	1.00

lower and upper bounds:

$$1 \text{ mm} \leq h_i, w_i \leq 50 \text{ mm}$$

Conclusion / Outlook

- Optimal control and design for underactuated systems
 - Provides the best possible structure and the corresponding control within a single optimization
 - Discrete control parameterization via cubic spline representation
 - ANCF formulation to describe large deformations and rotations
- Minimization of the moving mass of a flexible two arm manipulator
 - Scaling of nonlinear constraints due to different magnitudes
 - Solved by a direct optimization method with numerical gradients
- **Future tasks:**
 - Adjoint formulation for efficient gradient computation
 - Increasing the number of ANCF elements
 - Application to various examples, e.g. a swing up maneuver of a cart double pendulum with flexible links

